

THE RELATIONSHIPS BETWEEN STUDENTS' USES OF TECHNOLOGY, THEIR TASKS AND THE ARGUMENTS THEY CREATE

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Recent policy initiatives indicate that students should develop expertise in constructing viable arguments, engage in meaningful mathematical tasks, and strategically use appropriate tools, including technology. However, little research has been conducted that specifically examines the interplay between these three activities. The purpose of this paper is to describe the relationship between eighth grade students' uses of technology, the tasks in which they engage and how these relationships influence students' arguments. Findings indicate that specific types of tasks elicited certain uses of technology, and there seemed to be a relationship between the use of technology and the structure of the argument.

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The National Council of Teachers of Mathematics (NCTM) (2000) states that reasoning and proof are “fundamental aspects of mathematics” (p. 56) and “technology is essential in teaching and learning mathematics” (p.24). The Common Core State Standards for Mathematical Practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) agrees with NCTM indicating that students should develop expertise in constructing viable arguments and strategically use appropriate tools, including technology. Both organizations would agree that students should be given the opportunity to engage in activities in which they create and critique mathematical arguments and effectively use technology. Research has shown that the activities and tasks on which students work influences what they can learn (e.g. Stein, Engle, Smith & Hughes, 2008). Research has also indicated that the nature of a task may influence the use of technology and the structure of the argument (e.g. Hollebrands, Conner & Smith, 2010). However, research has not been conducted that specifically examines the interplay between argument, task and technology. The purpose of this paper is to describe these relationships for eighth grade students investigating properties and theorems associated with triangles while working in a dynamic geometry environment (DGE).

To briefly outline our investigation, we used Toulmin's (1958/2003) model of argumentation to examine students' mathematical arguments, which has been used by several researchers (e.g., Hollebrands, Conner & Smith, 2010; Lavy, 2006; Stephan & Rasmussen, 2002) in mathematics education. Once the arguments were diagrammed, we classified the students' uses of technology and the task on which they are working. For task, we coded each argument using the Mathematical Task Coding Instrument (MaTCI) (Heid, Bloom, Hollebrands, & Piez, 2003). To code for use of technology, we developed a list of codes based on the ways in which the students were using technology while engaged in tasks. Once coding was complete, we analyzed the data looking patterns and themes that emerged from the argumentation episodes, the types of tasks on which they were working and their use of technology.

Methods

As part of a larger study, the lead author conducted a two-week classroom teaching experiment with one eighth-grade mathematics class with the lead author serving as the

instructor. The study was conducted with students of varying ethnicities and socio-economic statuses at an urban public middle school in the southeast United States. The focus of the unit was the development of students' understanding of triangles, their properties, and associated theorems. For this class, technology played an integral role. Students used a DGE, The Geometer's Sketchpad (Jackiw, 2001), to explore and investigate concepts.

Participants

During the study, the instructor placed the students in pairs so they have the opportunity to have mathematical discussions with their partners while working on the tasks. These discussions were the primary focus of the study's analysis. Pairs were chosen rather than larger groups to maximize the opportunities for students to interact with the mathematical task while still having peer-to-peer discourse. From these pairings, four were purposefully selected to be the focus of the data collection based on recommendations made by the regular classroom teacher regarding their willingness to verbalize their thinking and ability to work well together. Of these four, three pairs (Amy and Judy; David and Erica; and, Heather and Mary) (all names are pseudonyms) were selected for analysis based on attendance.

Data Collection and Analysis

Data collection consisted of video and audio recordings, screen-capturing software that recorded students' uses of technology, and artifacts including students' written classwork, homework, quizzes, and exams. Of the eight class meetings, we analyzed the small group and whole-class discussions that centered on three topics (the triangle inequality theorem, the relationship between the magnitude of the side lengths and the measures of the opposite angles, and the properties of different types of triangles). The lead author created transcripts based on the audio and video recordings of the whole class discussions and the discussions for each of the three pairs of students during small group work. We used these transcripts, along with the artifacts, to identify reasoning episodes.

Using these data, we proceeded to make use of Toulmin's (1958/2003) model of argumentation to analyze and diagram students' arguments. Toulmin decomposed an argument into six components: claim (the conclusion whose validity is being established), data (the facts being appealed to as the foundation of the claim), warrant (the link between the data and the claim), backing (circumstances in which the warrant would otherwise be invalid), qualifier (confers the strength of the warrant), and rebuttal (circumstances in which the warrant does not hold). Figure 1 shows how these six components (data, claim, warrant, backing, qualifier, and rebuttal) fit together. Data is provided or constructed and "so" a claim is made based on this data. This claim can be made based on this data "since" the warrant. The warrant is relevant "on account" of the backing. The claim is valid "unless" the rebuttal occurs.

To identify an episode of argumentation, we looked for a student's verbal claim. After identifying each episode, we created a description of the argumentation for that claim which included the participants' words (from the transcripts) and actions including the students' gestures and uses of technology. Then, we diagrammed the argument according to the model developed by Toulmin (1958/2003). Each argument consisted of data, claim, and warrant, and, if contributed or needed, backing, qualifier, and rebuttal. Many times, students did not explicitly provide each component and we made inferences based on their previous work and use of technology. In these cases, we noted the inference in the diagrams by placing a "cloud" around the inferred component. If the component was known or spoken, we used a "box" to outline it. We attributed each component to the students, the teacher, or some combination of the teacher and students.

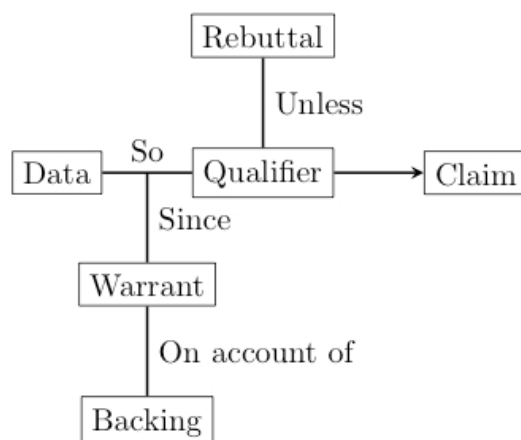


Figure 1. Toulmin's (1958/2003) Model of Argumentation

In order to determine a relationship between students' uses of technology and the task on which they are working, we first coded each argument using the Mathematical Task Coding Instrument (MaTCI) (Heid, Bloom, Hollebrands, & Piez, 2003) and then each task by the way in which the students were using technology. Heid and colleagues developed the MaTCI in order to "capture the sequence of tasks students took on as they worked with technology on conceptually oriented tasks" (p. 3). The authors designed this instrument to code tasks posed by the teacher, curriculum, and/or students. The task a student is working on may not always be the one assigned (Heid, et al., 2003). In fact, when students are working on a large task such as one posed by the curriculum, students often develop their own intermediate tasks to assist them in reaching their goal (Heid, et al., 2003). These intermediate tasks include developing and testing conjectures, producing intermediate data, describing what they see on their screen, and interpreting and justifying their results from their uses of technology. The focus of our coding was on the intermediate tasks that may or may not be posed by the curriculum. As shown in Table 1, the MaTCI framework accounts for these types of tasks by providing two levels of coding; categories and, within each category, subcategories. MaTCI was initially developed based on students' work with functions and algebraic tasks. Although the categories are general, the given explanations and examples are specific to this content. Thus, we modified the explanations to account for our students' work in geometry, specifically triangles.

For each episode of argumentation, students are developing claims based on data they collected. Thus, they are performing actions to accomplish a goal or, in other words, completing a task. Therefore, we coded the task for each argument using the Level 2 categories of the MaTCI framework. Many times, the students switched tasks within a single episode of argumentation. In these instances, we coded the additional task(s) using the same Level 2 categories of the MaTCI framework. For each argumentation episode, two researchers coded the same set of tasks individually and then compared their codes and resolved any conflicts by agreeing to a single code.

Table 1: Adapted MATCI (Heid, et al., 2003) Framework

Level 1 Category	Level 2 Category	Explanation
Concept	Identify	The task is to identify the name of an object when the characteristics of that object or an image of that object are presented to the student
	Describe	The task is to describe what you see (visual to perception) describing or characterizing a mathematical object.
	Elaborate	The task is to extend a previously stated idea by refining, specifying or clarifying it.
Product	Produce	The task is to create an object given a set of input values.
	Generate	The task is to create a rule or procedure for a particular problem/example/instance.
	Predict	The task is to describe what might happen under certain conditions in a novel situation. Students are asked to come up with a conjecture.
	Generalize	The task is to describe a relationship across multiple problems/examples/instances that holds for an entire class.
Reasoning	Corroborate	The task is to provide additional evidence that what is given or found is true. [Determine if something might be true or false].
	Justify	The task is to provide a logical argument for why something happens.

Once the initial and secondary tasks were coded, we proceeded to code each task in terms of the students' use of technology. For each task code, we examined the ways in which students used the technology while engaged in the task. We chose to use the task rather than the episode of argumentation as our unit of analysis because an episode may contain multiple tasks. Thus, there is potential for multiple uses of technology within a single episode as well. We developed these technology codes (see Table 2) based on our observations and revisited and refined them using a constant comparative method as described by Glaser (1965). Finally, we recorded each of task and technology codes in a spreadsheet and we used pivot tables to look for patterns and themes between mathematical tasks and technology use.

Table 2: Students Use of Technology Codes

Category	Explanation
Drag	Episodes where the use of the drag feature was used in the argument.
Appearance	Episodes where assumptions were based solely on visual aspects of the diagram
Measures	Episodes where numbers resulting from the use of commands in the "Measure" menu were used in an argument
Drag/Measures	Episodes where the use of the drag feature in conjunction with measures being displayed on screen were used in the argument
Imagined Dragging	Episodes in which the imagined movements of a figure were used in the argument
None	None of the above
Constructing	Setting up the problem situation such as dragging (adjusting) sliders

Results

The results reported in this paper are based on data taken from activities in which the goal was for students to develop an understanding of mathematical concepts and theorems related to triangles. The students used pre-constructed sketches to explore these concepts. While working on these tasks, the three pairs of students created arguments of various structures, engaged in a myriad of tasks, and used the technology in different ways. Four relationships emerged in our analysis of the students' use of technology and the tasks on which they were working.

Predict-Appearance

In over half (17/31) of the argumentation episodes with an initial task code of *predict*, the students' use of technology was the *appearance* of the figure on the screen. When students made predictions, they were not actively using the technology other than as a referent. For example, David and Erica were working on the triangle inequality task and were using the pre-constructed sketch to create segments of lengths 3, 4, and 4. Erica adjusted the sliders on the screen to create the segments, but she did not drag the segments to determine if a triangle was formed. David exclaimed, "Of course it's not going to work." This argument is illustrated in Figure 2.

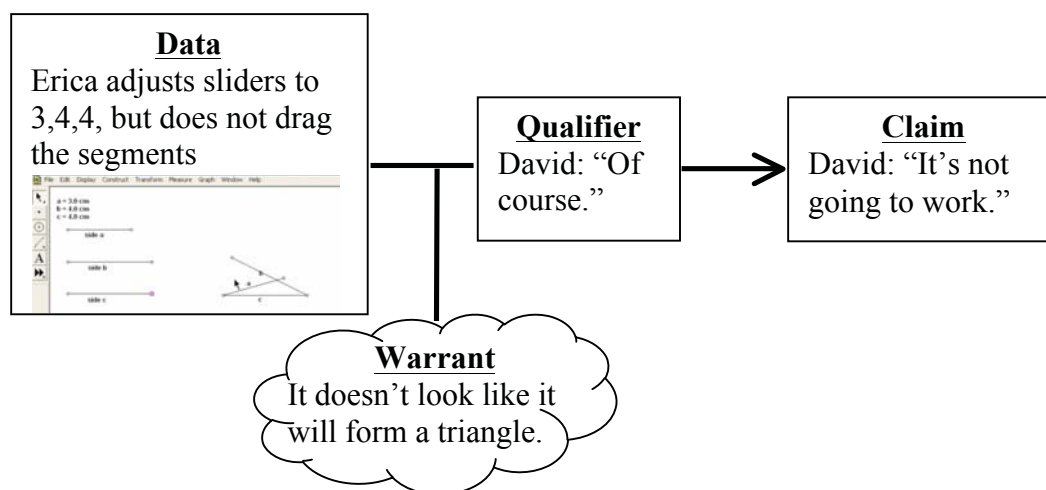


Figure 2: David and Erica's Argument with predict task and appearance

In this argument, David *predicted* whether the segments of lengths 3, 4, and 4 would form a triangle. His claim was based on the *appearance* of figure on the screen because Erica had yet to drag the segments to form a triangle prior to David's claim. David did not provide an explicit warrant for his claim, but we infer that his reasoning was based on the fact that the figure on the screen did not look like it would form a triangle.

Describe-Measure

At times, students created arguments as they provided *descriptions* of what they saw on the screen. During these episodes, the students would often use the *measurement* feature to provide a more accurate description of the figure. For example, David and Erica were determining the longest side of obtuse triangle ABC . On his activity sheet, he wrote that AB is the shortest side. Using the technology, David previously measured the side lengths to be $mAB = 4.90$ cm, $mBC = 2.54$ cm, and $mAC = 3.01$ cm. Erica displayed the obtuse triangle on the screen, which included the measures of the lengths of the sides. Erica exclaimed, "No it's not. That's wrong. That's

wrong.” She told David, “Look at the measurements... AC .” David affirmed this claim stating, “That’s right.” This argument is illustrated in Figure 3

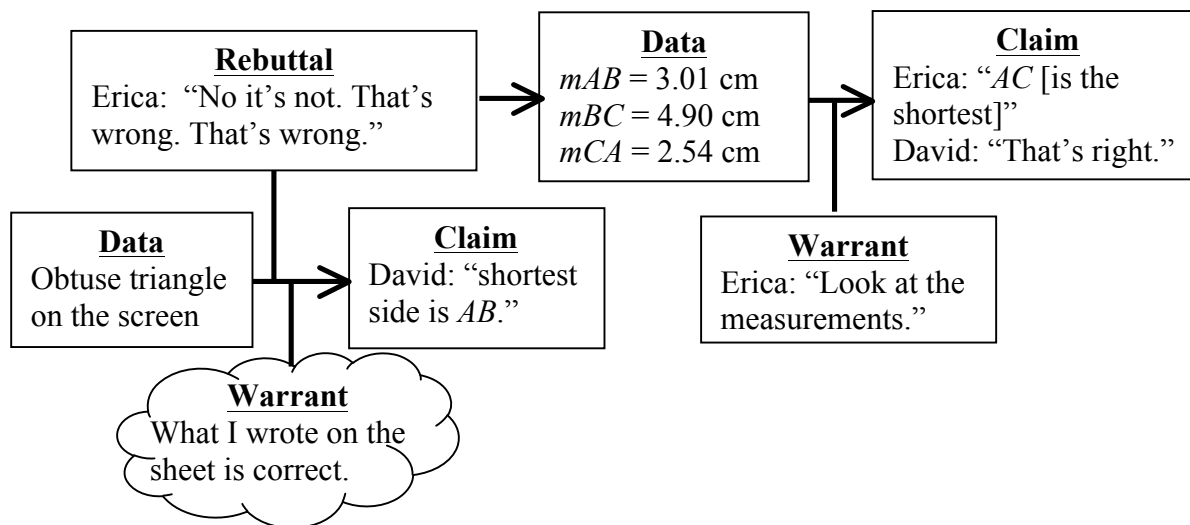


Figure 3: David and Erica’s Argument with Describe Task and Measures

We coded the task for this episode as *describe*. David and Erica were not producing a triangle nor were they attempting to classify the triangle. Rather, they were providing a description of the relative lengths of the sides. To make this description, the students used measures. As students became more comfortable with the technology, they tended to stop using the measure feature and began making claims based on the appearance of the figures. Thus, when students were working on description tasks, their use of technology shifted from relying on measures to basing their claims on appearance. However, the students would use measures if prompted to do so.

Produce-Drag

Many of the tasks the students worked on when using the technology were *produce* tasks. In these tasks, students attempted to produce a figure based on certain criteria. The teacher may have given the criteria to the students via the activity sheet, general instructions to the class, or specific instructions to the pair of students. The students, at times, generated their own criteria to produce a figure. While producing the figure, the students often made use of the *drag* feature. For example, Heather and Mary were determining whether segments of lengths 3, 4, and 4 would form a triangle (similar to Erica and David in the argument illustrated in Figure 1). Heather adjusted the sliders accordingly and dragged the endpoints. As Heather dragged an endpoint of the figure and was unable to immediately form a triangle, Mary stated, “No [it will not form a triangle].” Heather replied, “No it’s too long. But that’s weird, wouldn’t b be able to reach 3?” She continued to drag the endpoints and was able to form a triangle. She exclaimed, “Wait, wait, I got it.” In this argument, Heather and Mary were attempting to produce a triangle with side lengths 3, 4, and 4. To accomplish this task, Heather used the drag feature of the technology.

Generalize-None and Justify-None

When students worked on tasks in which they were creating *generalizations* or *justifying* their ideas, they generally did not actively use technology. Instead, the students would often appeal to mathematical definitions, theorems, and formulas; patterns they noticed in their

recorded data; or, the authority of another individual. For example, one of the arguments common to all pairs of students in structure was in response to the question on the activity sheet, “Why was it impossible to construct a triangle with some of the given lengths?” One student, Erica, responded, “One’s [segment] too long or too short.” This argument is illustrated in Figure 4a. The question asked the students to generalize across the examples. The data used by the pairs of students to support their claims were their answers to the examples sets of segments on their activity sheet. To gather this data, the students used technology. However, when responding to this question, the data had been previously collected and their reasoning was not based on their active use of technology, but on the product of their previous uses. Thus, Erica did not actively use technology in this argument.

Similar to generalization tasks, students rarely used technology when engaged in justification tasks. In fact, of the seven episodes coded justify, six did not include the use of technology. For example, the teacher posed the question, “Can we have two right angles in a triangle?” The class, including Judy, responded, “No”. When asked for her reasoning, she simply stated, “It would look like a square.” This argument is illustrated in Figure 4b. To answer and justify her response to the teacher’s question, Judy did not use technology.

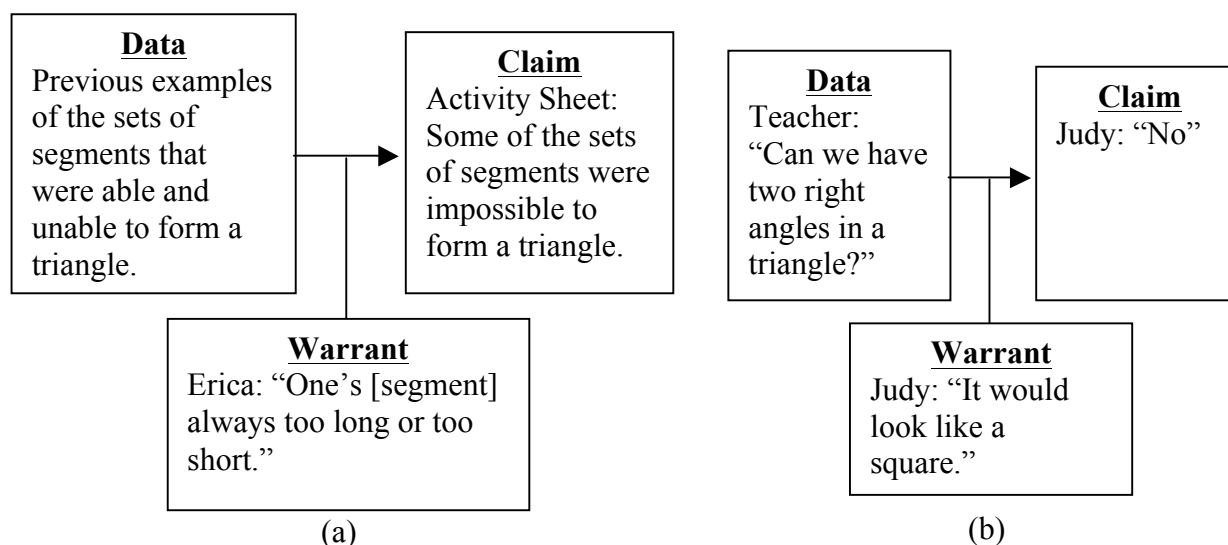


Figure 4: Arguments in which students did not use technology and engaged in (a) generalize tasks and (b) justify tasks.

Discussion

As the results of this study indicate, relationships exist between the ways in which students use technology when engaging in particular tasks. Specifically, students use the *appearance* of the figure when engaged in *predict* tasks, use the *measurement* feature when working on *describe* tasks, use the *drag* feature to *produce* a new figure, and do not use technology when *generalizing* or *justifying*. Hollebrands, Conner and Smith (2010) had a similar finding in their study of the arguments college geometry students created when working with technology. When the college geometry students provided explicit warrants for their claims, the students were generally not using technology and they were working on proofs, a particular kind of justification tasks. The authors attributed this finding to the students’ prior experiences in learning mathematics at the collegiate level where the students were expected to provide formal

proofs, which did not involve the use of technology. We cannot do the same to the middle school students in this study because it is unlikely they had been exposed to formal proofs. Rather, their lack of use of technology while engaged in these types of tasks may be due to their inexperience in using technology when working on these types of tasks.

As previously mentioned, when students were not actively using technology, they were mainly working on *generalization* and *justification* tasks. However, other researchers (e.g. Healy & Hoyles, 2001) found that students will create generalizations while using technology. Some pairs of students in Healy and Hoyle's (2001) study used a DGE to investigate relationships among the angle bisectors of a quadrilateral. The pairs of students did not create the same constructions and did not arrive at the same conclusions. However, those students that were successful were able to construct and measure aspects of their diagrams and developed generalizations while using the DGE. The students in the current study did not have the option of creating their own diagram. Instead, the students in the current study used a teacher-generated pre-constructed sketch that limited the students in how they could modify and/or measure aspects of the diagram. Perhaps, the students in the current study would have been more likely to use the technology while working on *generalization* tasks if they had been given the opportunity to create their own diagrams.

The results of this study should inform the design and implementation of activities that engage students in using technology to learn mathematics. Teachers should have a better understanding of the types of tasks in which their students are engaged by examining the ways in which their students are using technology. Teachers and teacher educators should also be aware that students are unlikely to use technology while engaging in *generalize* and *justify* type tasks. Teachers and teacher educators should design activities that encourage students to engage in these types of tasks with technology.

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